Phase Analysis of Lamb Waves by AP-Wavelet Transform

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Abstract. In recent years, factorization of Fourier spectrum is used for seismic wave analysis to separate the effects of source and propagation path. In the analysis, seismic waves are separated into amplitude-dependent parts, called minimum-phase functions (MPS), and the other parts, called all-pass function (AP). M. Tai et. al. [1], expanded this factorization algorithm to time-frequency analysis of wavelet transform.

On the other hand, wavelet transform is widely used for analysis of Lamb wave AE. For example, a number of researchers use wavelet transform to obtain group velocity dispersions of Lamb waves AE. As a phase spectrum of signal change not only by pure phase shift, but also by amplitude spectrum changing, we extracted pure phase shift by factorization using wavelet transform (AP-wavelet transform).

The effectiveness of AP-wavelet was evaluated by applying the simulated and measured dispersive signals. The results show the possibility to extract further information when applying AP-WT compared to applying standard wavelet transform.

1. Introduction

Lamb waves are elastic waves which propagate in thin structures. As Lamb waves propagate over long distance with small attenuation, the waves are frequently used for non-destructive testing of large engineering structures. As phase and group velocities of Lamb waves are changed with frequency and thickness of the materials, the waves show dispersion nature. To evaluate dispersion nature of Lamb waves, group velocity dispersion is frequently observed by using wavelet transform. On the other hand, phase information of Lamb waves is not effectively used for the Lamb wave analysis.

In general, phase spectrum contains the information of arrival time delay of each frequency component of Lamb waves. Then, there is a possibility to utilize the phase information of Lamb wave AE for AE analysis. Since phase spectrum obtained by general signal processing contains components corresponding to amplitude spectrum, these components should be eliminated. We eliminated this components by the signal processing of the spectrum factorization based on the Kramers-Kronig (K-K) relation and decomposed the received signal into a pair of functions, so called minimum phase shift (MPS) and all pass (AP) functions. The AP function has no information on its amplitude spectrum, however, it stores the information of phase spectrum of the received signal. So, we call the wavelet transform using the AP function as AP-WT in this paper.

2. Spectral factorization

As the signals propagated through the transfer function $H(\omega)$ which satisfies the causality such as seismic wave, the relationship between the real part and the imaginary part of the transfer function is described by the Hilbert transform as equation (1).

$$\mathrm{Re}[H(\omega)] = \mathrm{HT}[\mathrm{Im}[H(\omega)]] = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{\mathrm{Im}[H(\omega')]}{\omega' - \omega} d\omega' \tag{1}$$

The P.V. in the equation means the Cauchy principal value. This relationship is called the Kramers-Kronig (K-K) relation. K-K relation can be also applied to the relation between the amplitude and phase spectrum of the causal transfer function, because the logarithm of the complex transfer function is a sum of the real part as logarithm of amplitude spectrum and the imaginary part as the phase spectrum in equation (2).

$$H(\omega) = |H(\omega)| \exp\{j\varphi(\omega)\}, \ \ \ln H(\omega) = |\ln H(\omega)| + j\varphi(\omega) \tag{2}$$

Note that this expression exists with the assumption that there is no zero point in the amplitude spectrum.

The Lamb waves are also supposed to satisfy the causality, and the phase spectrum derived from the amplitude spectrum of received signal can be calculated using K-K relation. The set of these amplitude and phase spectrum is called a minimum phase shift function (MPS) $F_{\mathrm{MPS}}(\omega)$ which is determined by the amplitude spectrum of the signal $S(\omega)$ as equation (3), and represents the liner transfer function.

$$F_{\mathrm{MPS}}(\omega) = |S(\omega)| \exp\{j\mathrm{HT}[-\ln|S(\omega)|]\} \tag{3}$$

The phase spectrum obtained with dividing the Fourier spectrum of the signal by MPS function in the frequency domain as equation (4), is called all pass (AP) function which has no information in its amplitude spectrum.

$$F_{\mathrm{AP}}(\omega) = S(\omega)/F_{\mathrm{MPS}}(\omega) \tag{4}$$

In this way, we extracted pure phase shift information of the signal as the AP function, and applied to continuous wavelet transform. We called a series of this signal processing as AP-WT.

3. Comparing AP-WT with standard wavelet transform

3.1 Theoretical S_0 mode Lamb wave

To examine the advantage of AP-WT, we compare the result of AP-WT with that of standard wavelet transform by analysing the S_0 mode Lamb wave which is obtained by solving following S_0 mode Lamb wave equation.

$$u(x,t) = \int_{-\infty}^{\infty} G(\omega) e^{j\frac{\omega x}{C_p(\omega)}} e^{-j\omega t} d\omega \tag{5}$$

Here, x, $G(\omega)$, $C_p(\omega)$ are the propagation distance, the input signal, and the phase velocity respectively. In this research, we set these parameters as: x=100mm, $G(\omega)$ is the Gaussian pulse, and $C_p(\omega)$ is the situation of aluminium plate with 1mm thickness. The time-domain waveform of the signal and its amplitude spectrum is shown in Fig.1 (a) and (b).Using K-K relation, the time domain AP function is obtained as shown in Fig.1 (c).

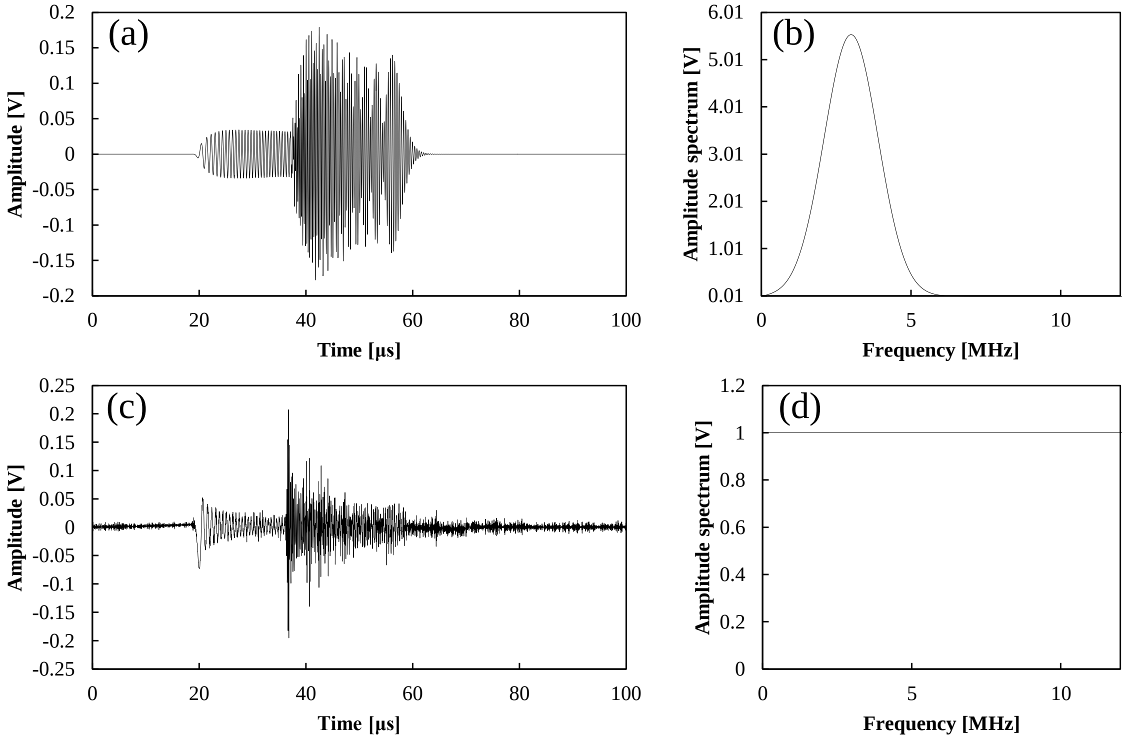


Fig. 1 (a) The time domain waveform of the signal for analysis and (b) the amplitude spectrum in frequency domain of the signal. (c) The time domain waveform of AP function of the signal and (d) the amplitude spectrum in frequency domain of AP function.

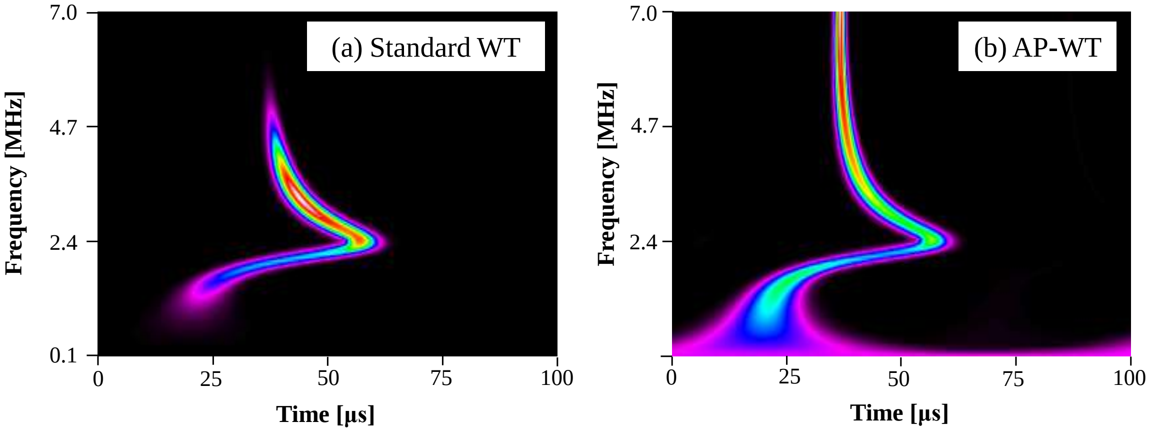
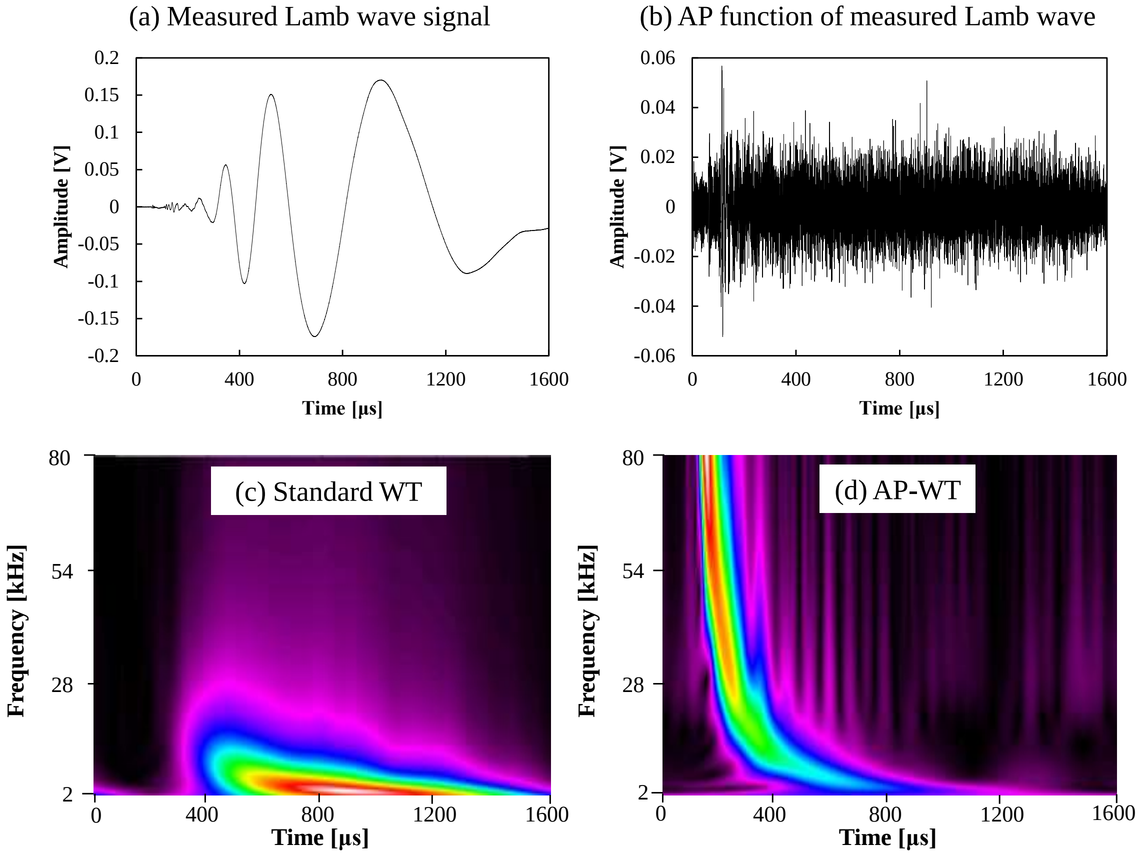


Fig. 2 Wavelet contour map of standard- and AP- WT. (a) wavelet transform with theoretical single S_0 mode wave ((a) in Fig. 1), and (b) wavelet transform with AP function of theoretical single S_0 mode ((c) in Fig.1).

The wavelet contour map (the colour in the map represents amplitude of wavelet transform) of signals shown in Fig.1 (a) and (c) are shown in Fig.2 (a) and (b) respectively. Both results represent the arrival times of group velocity of the S_0 mode, but AP-WT made the result clear in low and high frequencies region. This is because of the all pass characteristic of the AP function in frequency domain.

3.2 Experimental signal of Lamb wave

We performed the Lamb wave propagation experiment using the PMMA plate with 1mm thickness. The source was pencil-lead break and the receiver was located at 100mm distance from the source. The measured signal is shown in Fig.3 (a), and its AP function is shown in Fig.3 (b). The obtained AP function (b) seems to contain high frequency noises. Fig.3 (c) and (d) are wavelet contour maps of the signals shown in (a) and (b) respectively. In comparison to standard wavelet transform (c), the AP-WT is successful to extract dispersion curve of A_0 mode group velocity.


(a) Measured Lamb wave signal

(b) AP function of measured Lamb wave

Fig. 3 (a) Experimentally measured Lamb wave for PMMA thin plate, and (b) its AP function. (c) wavelet contour map of measured signal, and (d) its AP-WT.

4. Applications of AP-WT

4.1 Fracture modes classification of CFRP by Acoustic emission analysis

Acoustic emission (AE) testing is the effective tool for observing fracture process of Carbon fiber reinforced plastics (CFRP). AE signals produced by damages in a thin CFRP plate are propagate as dispersive Lamb waves and its characteristics are changed with CFRP fracture modes. In order to correlate observed AE with the fracture mode, it is necessary to emphasize the characteristics of AE signals by signal processing. Mizutani et. al. [2] detected AE during in-plane compression test of cross-ply CFRP. They detected AE signals produced by fiber fracture, transverse crack, delamination and matrix crack. They also simulated AE signals generated by four fracture modes by using pulse YAG laser.

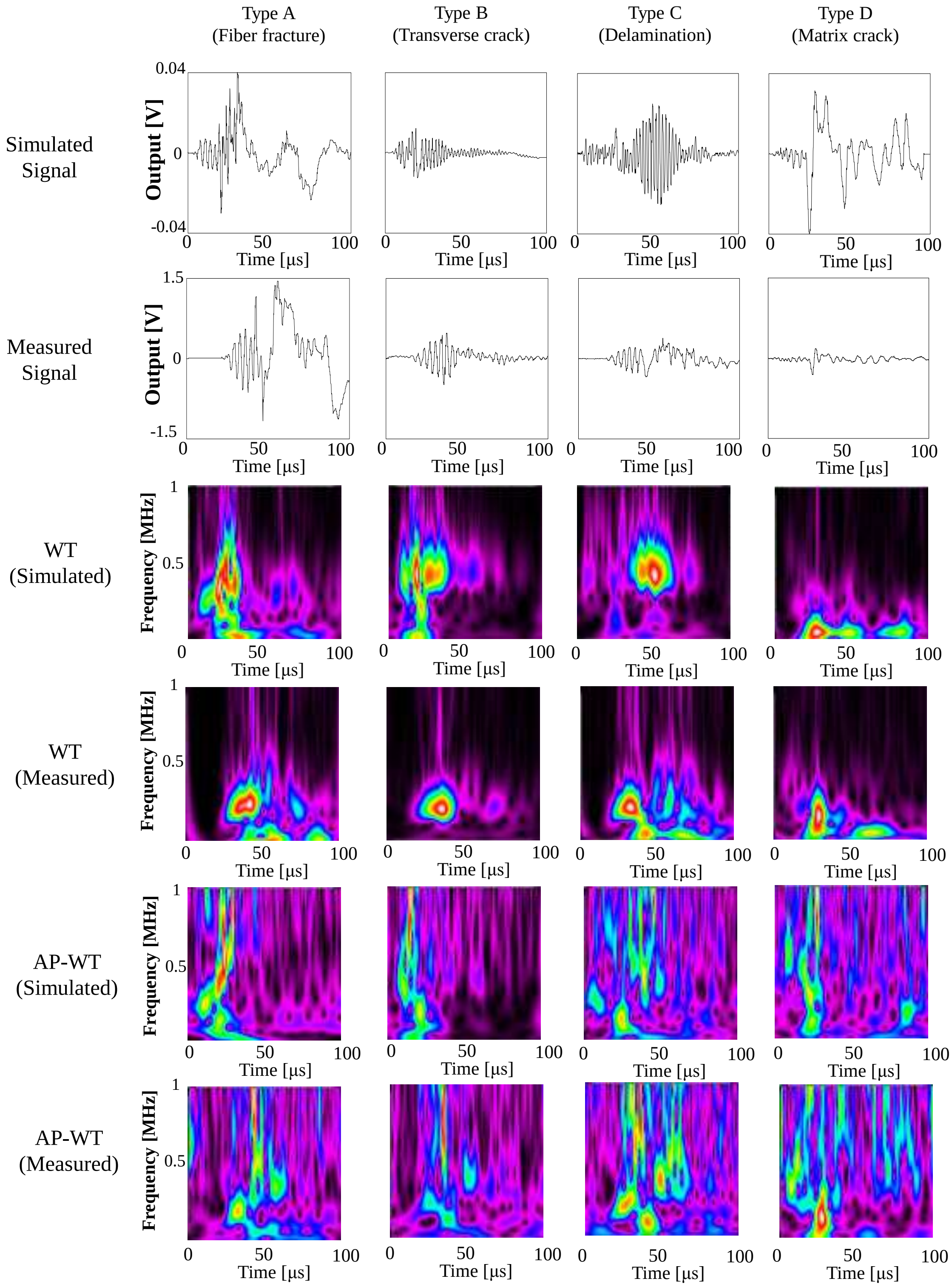


Fig. 4 AE caused by different fracture modes (upper) and their wavelet transform by standard WT and AP-WT (lower).

They compared characteristics of measured hundreds of AE with simulated four AE by using wavelet contour maps, and estimated fracture process of a CFRP plate.

In this paper, we substitute AP-WT for wavelet transform as the mean of comparing AE signals and classifying. The signal datasets are experimentally measured AE and simulated AE.

Results are shown in Fig.4 and lower 2 wavelet contour maps are obtained by AP-WT for each fracture type. The characteristics of time-frequency distribution in the AP-WT between simulation signals and measured signals resemble more than those of standard wavelet transform, especially in Type A (Fiber fracture). So, it is expected to classify AE signal types easily by using not only standard wavelet transform but also by the AP-WT.

4.2 THz electromagnetic signal processing

Although it is not signal processing of AE, the AP-WT can be applied to signal processing of other non-stationaly time-domain waveform such as electromagnetic waves. Terahertz (THz) waves is electromagnetic waves within the band of frequencies from 0.3 to 3 terahertz, and its wavelengths range from 1 mm to 0.1 mm. Frequency band of THz wave exists between microwaves and infrared light waves, but THz waves can be handled in time-domain recording.

We performed the experiment to examine the ability of detecting microstructure using THz pulse wave as shown in Fig. 5. THz pulse waves were irradiated toward cube protrusions (size : 0.3 to 0.5 mm) on the aluminium plate, and received THz signals were analysed by standard wavelet transform and the AP-WT. The result is shown in Fig.6. The AP-WT could detect the influence of protrusions as time delay in specific frequencies. Because the smaller size of protrusions, the higher frequencies component arrives late, this result is assumed to represent diffraction phenomenon of THz pulse waves. From this result, it is assumed that the AP-WT has possibility of extracting characteristics of signals clearly in other time domain signals.

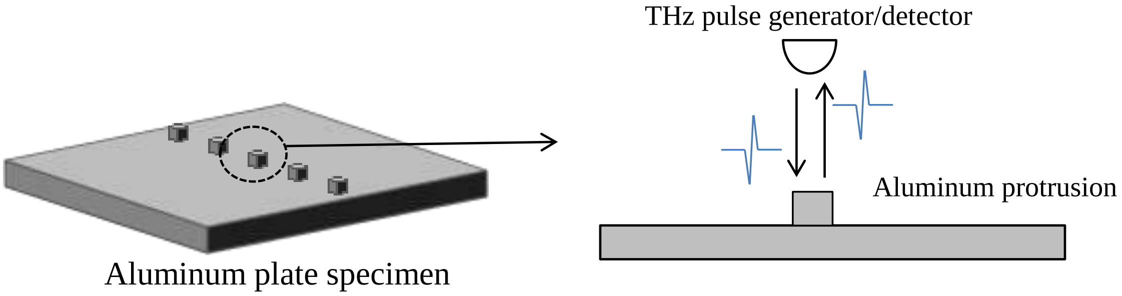


Fig. 5 Experimental set up for THz pulse reflection. Cube protrusions were located on the aluminium plate to investigate influence on received THz pulse waves.

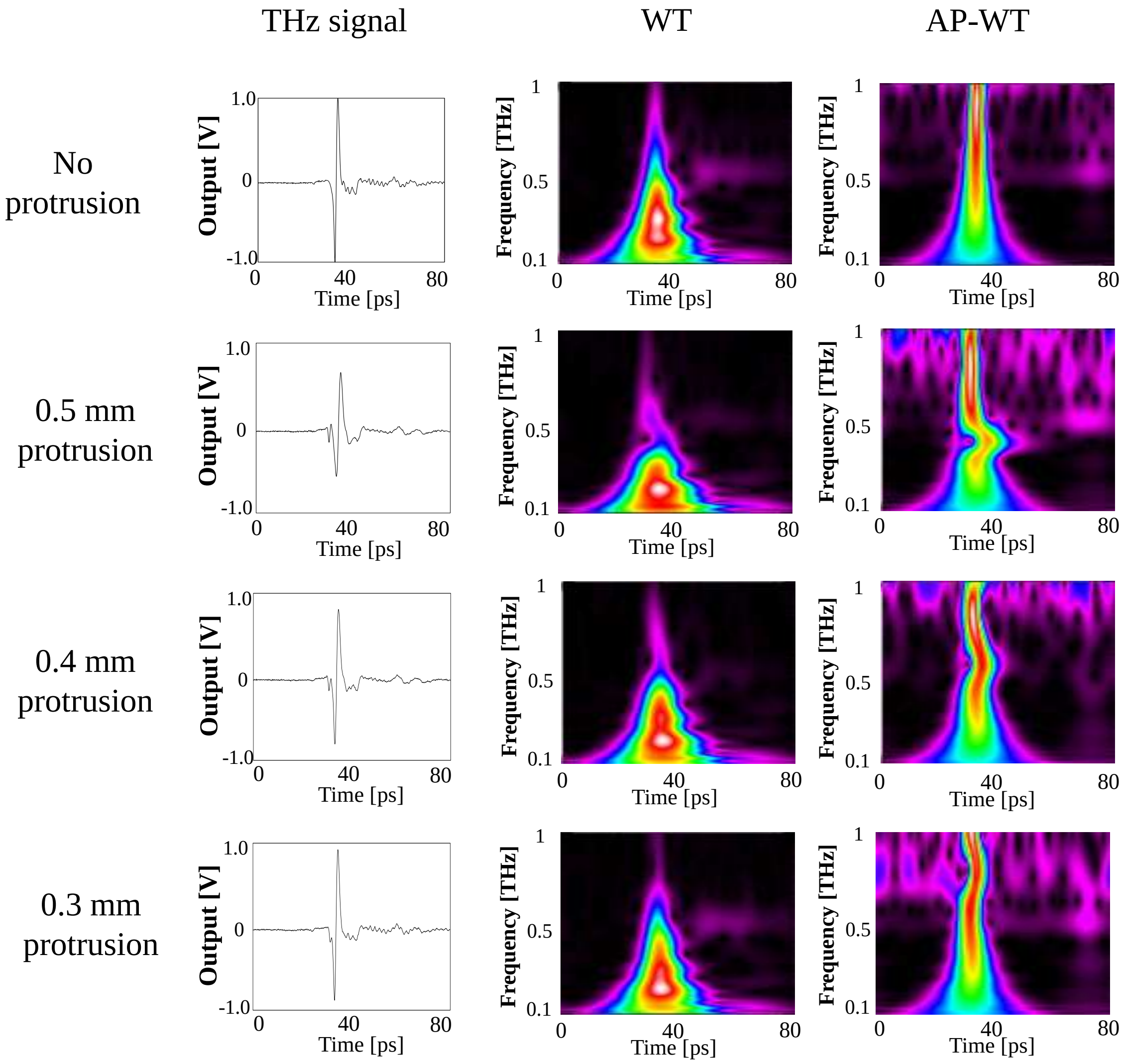


Fig. 6 Four received THz pulse signals from each size of protrusion. Results of AP-WT were successful to extract arrival time delay in specific frequencies corresponding to size of protrusions.

5. Conclusion

AP function calculated by spectral factorization of Lamb waves signals based on Kramers-Kronig relation was applied to continuous wavelet transform (AP-WT). Since AP function has no amplitude information of signals, phase shift information could be extracted by wavelet transform. With AP-WT, dispersion curves of Lamb waves become clear comparing to standard wavelet transform. We next applied AP-WT to AE from CFRP and diffracted electromagnetic waves from cube protrusions on a plate. The results show the possibility to extract further information when applying AP-WT compared to applying standard wavelet transform.

References

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