Estimation of Viscoelastic Properties by Lamb Wave Analysis

Yoshihiro MIZUTANI *, Kazuhiro SUENAGA *, Akira TODOROKI *,
Yoshiro SUZUKI *
* Tokyo Institute of Technology, 2-12-1-I1-70, Ookayama, Meguro-ku, Tokyo, Japan

Abstract. Engineering plastics are widely used as structural members. As viscoelastic properties are changed with age of service and usage environment, it is important to measure the properties periodically with non-destructive way. In this study, thin walled structures made of engineering plastics were focused. Several AE sensors were mounted on the thin plates, and then elastic waves were generated by steel ball impact. As elastic waves propagate as Lamb wave in thin walled structures, we estimated viscoelastic properties from Lamb wave analysis. In order to estimate the properties accurately, accurate measurement of attenuation and velocity of Lamb waves are required. To realize accurate measurement, signal processing methods used for AE testing were utilized. For accurate attenuation measurement, the way of sensitivity calibration for AE sensors were utilized. On the other hand, source location algorithm was used for accurate velocity measurements. Finally, viscoelastic properties of 0.4 mm thickness PMMA were successfully measured by the proposed method.

1. Introduction

In recent years, engineering plastics are widely used for thin-walled structures of transportation vehicles. For such parts, one of the important issues is quantitative evaluation of degree of degradation. The evaluation can be achieved when the viscoelastic properties of the thin plastic structures can be measured in non-destructive ways.

On the other hand, Lamb waves are known as the elastic waves propagate in a thin medium. As group velocity and attenuation ratio of the waves are affected by viscoelastic properties of the propagation medium, the viscoelastic properties can be estimated by analysing the lamb waves [1-5]. For the estimation, however, accurate velocity and attenuation of Lamb waves are required. In this study, we proposed new method for achieving accurate measurements of both the velocity and attenuation by utilizing techniques which are used in AE monitoring fields.

2. Experimental setup

Figure 1 shows the experimental setup. A 0.4 mm-thickness PMMA plate (550mm^{W}×400mm^{H}) is prepared for this experiment. Four AE sensors (Physical Acoustics, Type: PICO) are attached on the plate as in the figure. Lamb waves are generated by impact of dropping steel balls (diameter: 4.8 mm) on the plate and detected by four AE sensors (A~D in Fig. 2). Desired source locations of Lamb waves (Impact points of steel balls) are shown in Fig. 2. Figure 3 shows examples of Lamb waves detected by sensor A when the AE is generated at (x,y) = (30mm,70mm).

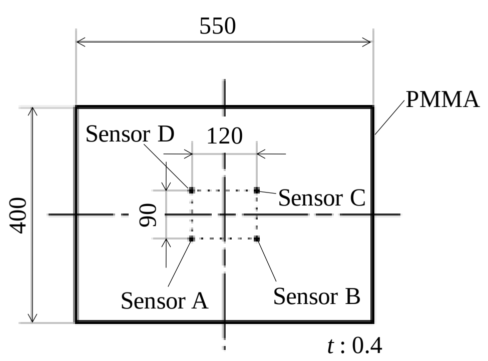


Fig. 1. Schematic illustration of experimental setup.

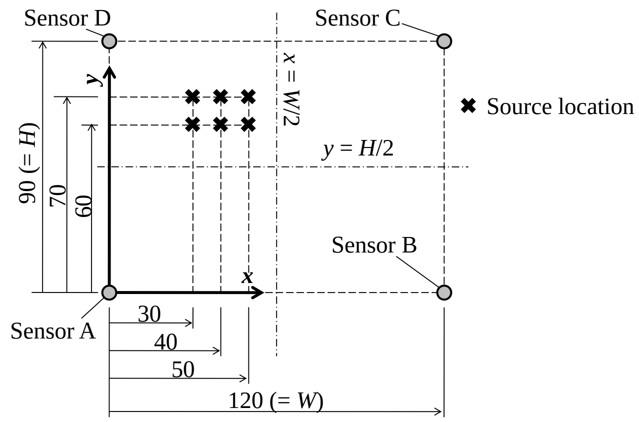


Fig. 2. Source location of Lamb waves.

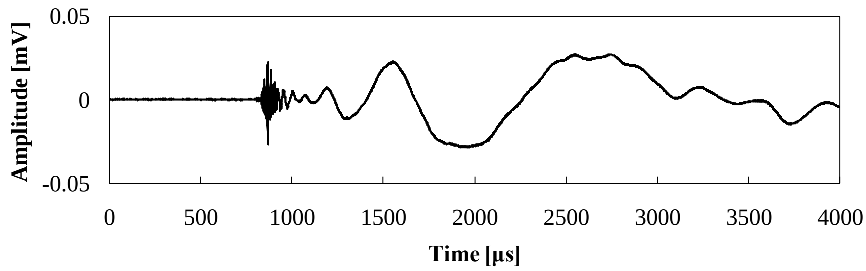


Fig.3. Examples of Lamb wave detected by sensor A. (Source location: x=30mm, y=70mm.)

3. Accurate measurement of group velocity

To achieve accurate group velocity $c_g(\omega)$ measurement, source location (x,y) need to be measured accurately. Following source location calculation used in AE monitoring field is utilized in this study.

$$x = \frac{W^2 - c_g^2(\omega)\cdot\left(\Delta t_B^2(\omega) - \Delta t_A^2(\omega)\right)}{2W} \tag{1}$$

$$y = \frac{H^2 - c_g^2(\omega)\cdot\left(\Delta t_D^2(\omega) - \Delta t_A^2(\omega)\right)}{2H} \tag{2}$$

$$\Delta t_X(\omega) = t_X(\omega) - t_0(\omega) \quad (X = A, B, C, D) \tag{3}$$

$$c_g^2(\omega) = \frac{-b(\omega) \pm \sqrt{b^2(\omega) - 4a(\omega)\cdot c}}{2a(\omega)} \tag{4}$$

$$a(\omega) = H^2\left(\Delta t_B^2(\omega) - \Delta t_A^2(\omega)\right)^2 + W^2\left(\Delta t_D^2(\omega) - \Delta t_A^2(\omega)\right)^2 \tag{5}$$

$$b(\omega) = -2(WH)^2\left(\Delta t_B^2(\omega) + \Delta t_D^2(\omega)\right) \tag{6}$$

$$c = (WH)^2\left(W^2 + H^2\right) \tag{7}$$

Here, x and y are the source location of Lamb wave AE, $t_0(\omega)$ is generation time of Lamb wave, $c_g(\omega)$ is group velocity of each frequencies. Definition of W and H are shown in the Fig. 2. Arrival times of Lamb waves for each sensor $t_X(\omega)$ $(X = A, B, C, D)$ are determined by using wavelet transform. Source locations (x and y) and group velocities ($c_g(\omega)$) are determined simultaneously by the equations (1)-(7) from the measured $t_X(\omega)$. Source locations can be estimated by certain frequency ω, although, source location should be independent from ω. By utilizing this fact, estimated source location can be corrected by using a number of data set with different frequencies ω. Figure 4 and 5 shows estimated source locations and group velocities. Dotted points ● in Fig. 4 show aimed impact points and squares ◊ show estimated source locations. Six curves in Fig. 5 show the group velocity dispersions estimated by Lamb waves induced at the positions *x* and *y*. As velocities increase with frequency, measured velocities should be A_0-mode Lamb waves. Measured group velocity dispersions are used for estimating viscoelastic properties of plastic plates.

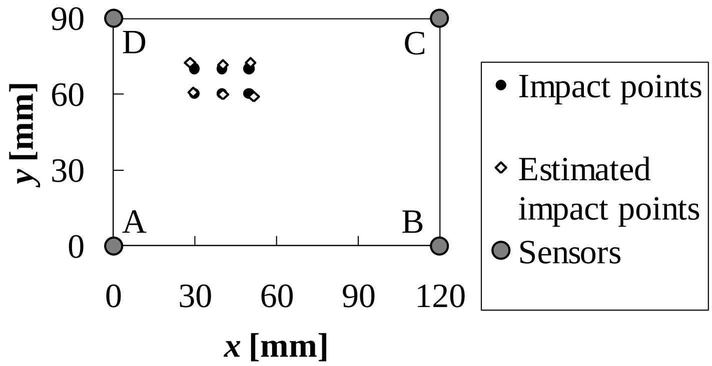


Fig. 4. Estimated source location.

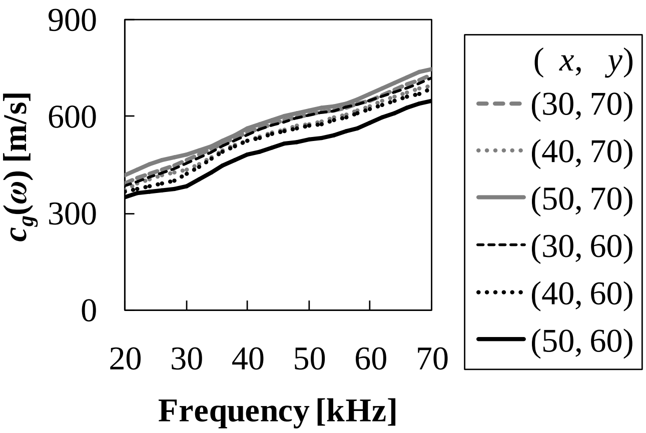


Fig. 5. Measured group velocities $c_g(\omega)$ of Lamb waves generated by steel ball impacts on PMMA plate.

4. Accurate measurement of attenuation ratio

In this chapter, the method which achieves accurate attenuation ratio measurement is proposed. In general, decay of the amplitude of Lamb waves with propagation distance *r* can be expressed as following equation [6].

$$A(\omega, r) = A_0(\omega)\frac{1}{\sqrt{r}}\exp(-\alpha(\omega)\cdot r) \tag{8}$$

Here, *A* is the amplitude of Lamb waves at propagation distance *r* and frequency $\boldsymbol{\omega}$, A_0 is the amplitude at the source position, $\boldsymbol{\alpha}$ is attenuation ratio. The equation takes into account both the absorption and scattering of the waves [7].

When considering sensitivities of each sensor $S_X(\boldsymbol{\omega})$ $(X = A, B, C, D)$ and cyclic excitation of Lamb waves, following equation can be derived from equation (8).

$$A_{iX}(\omega) = S_X(\omega) A_{0,i}(\omega) \frac{1}{\sqrt{r_{iX}}} \exp(-\alpha(\omega) \cdot r_{iX}) \tag{9}$$

$$(i = 1 \sim 6, \quad \mathrm{X} = \mathrm{A} \sim \mathrm{D})$$

Here, A_{iX} is the amplitude of Lamb waves of i^{th} experiment at the sensor position of X. $A_{0,i}$ is the amplitude of the Lamb waves at the source position which is excited at i^{th} experiment. r_{iX} is propagation distance from source to sensor X at i^{th} experiment. The r_{iX} can be determined acculately by using the method which is explained in the previous chapter. As linear relationship between amplitude of Lamb waves and scalogram of wavelet transform is assumed, we substitute measured maximum wavelet scalogram of each frequency $\boldsymbol{\omega}$ to A_{iX} to obtain attenuation ratios. In this study, Lamb waves were excited 6 times (i=0~6). Following equation (10) can be derived from equation (9).

$$\ln(A_{iX}(\omega)) + \frac{1}{2}\ln(r_{iX}) = -\alpha(\omega) \cdot r_{iX} + \ln(S_X(\omega)) + \ln(A_{0,i}(\omega)) \tag{10}$$

In order to obtain $\boldsymbol{\alpha}$, S_x and $A_{0,i}$ should be eliminated from the equation. To eliminate these parameters, following subtraction is conducted.

$$\begin{aligned} &\left[\left\{\ln(A_{jY}(\omega)) + \frac{1}{2}\ln(r_{jY})\right\} - \left\{\ln(A_{jZ}(\omega)) + \frac{1}{2}\ln(r_{jZ})\right\}\right] \\ &- \left[\left\{\ln(A_{kY}(\omega)) + \frac{1}{2}\ln(r_{kY})\right\} - \left\{\ln(A_{kZ}(\omega)) + \frac{1}{2}\ln(r_{kZ})\right\}\right] \\ &\qquad = -\alpha(\omega) \cdot \{(r_{jY} - r_{jZ}) - (r_{kY} - r_{kZ})\} \end{aligned} \tag{11}$$

$$(j, k = 1 \sim 6,\ j \neq k, \quad \mathrm{Y}, \mathrm{Z} = \mathrm{A} \sim \mathrm{D},\ \mathrm{Y} \neq \mathrm{Z})$$

As A and r in the equation can be measured and estimated, the attenuation factor $\boldsymbol{\alpha}$ can be estimated by using least squares method. Figure 6 shows estimated results.

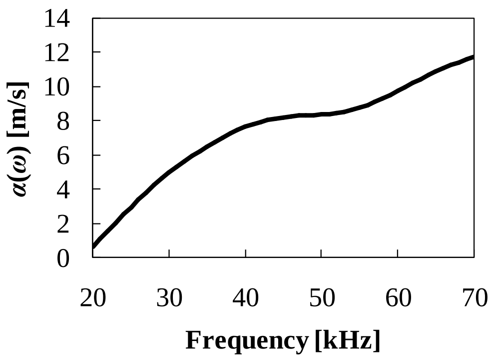


Fig. 6. Attenuation ratios of each frequency $\alpha(\omega)$ of Lamb waves measured in PMMA plate.

5. Estimation of viscoelastic properties

In this chapter, the method for estimating viscoelastic properties from C_g and $\boldsymbol{\alpha}$ is explained. It is well known that group velocity C_g and attenuation ratio $\boldsymbol{\alpha}$ can be calculated by using complex modulus of elasticity E^* of viscoelastic materials as shown in following equations [5].

$$k_L^{*2}(\omega, E^*(\omega)) = \omega^2 \frac{\rho}{E^*(\omega)} \frac{(1+\nu)(1-2\nu)}{1-\nu} \tag{12}$$

$$k_T^{*2}(\omega, E^*(\omega)) = \omega^2 \frac{\rho}{E^*(\omega)} \cdot 2(1+\nu) \tag{13}$$

$$\frac{\tan\left(q^*(\omega, E^*(\omega))\frac{d}{2}\right)}{\tan\left(p^*(\omega, E^*(\omega))\frac{d}{2}\right)} + \left\{\frac{4p^*(\omega, E^*(\omega))q^*(\omega, E^*(\omega))k_{Lamb}^{*2}(\omega, E^*(\omega))}{\{(q^*(\omega, E^*(\omega)))^2 - k_{Lamb}^{*2}(\omega, E^*(\omega))\}^2}\right\}^{\pm 1} = 0 \tag{14}$$

Here, $\{p^*(\omega, E^*(\omega))\}^2 = k_L^{*2}(\omega, E^*(\omega)) - k_{Lamb}^{*2}(\omega, E^*(\omega))$

$\{q^*(\omega, E^*(\omega))\}^2 = k_T^{*2}(\omega, E^*(\omega)) - k_{Lamb}^{*2}(\omega, E^*(\omega))$

$$c_{g,C}(\omega, E^*(\omega)) = \left\{\frac{d}{d\omega}\mathrm{Re}(k_{Lamb}^*(\omega, E^*(\omega)))\right\}^{-1} \tag{15}$$

$$\alpha(\omega, E^*(\omega)) = -\mathrm{Im}(k_{Lamb}^*(\omega, E^*(\omega))) \tag{16}$$

We first calculated k^*_{Lamb} by solving the equations (12) ~ (14). Then, C_g and attenuation ratio $\boldsymbol{\alpha}$ was calculated by substituting k^*_{Lamb} to equations (15) and (16). For the calculation of (13) and (14), we used following properties for the density and Poisson's ratio.

Table 1. Parameters substituted into equations (12), (13) in Complex modulus estimation.

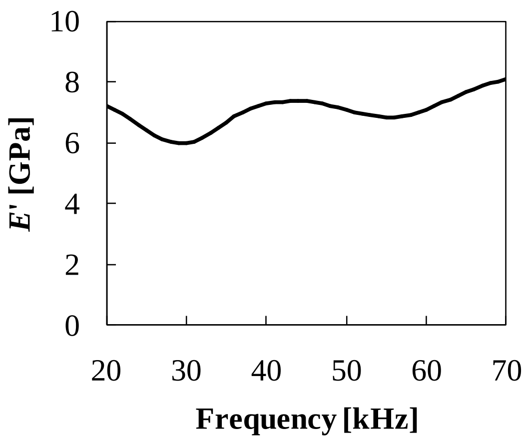
ρ [kg/m^3]	ν
1.2×10^3[8]	0.33[8]

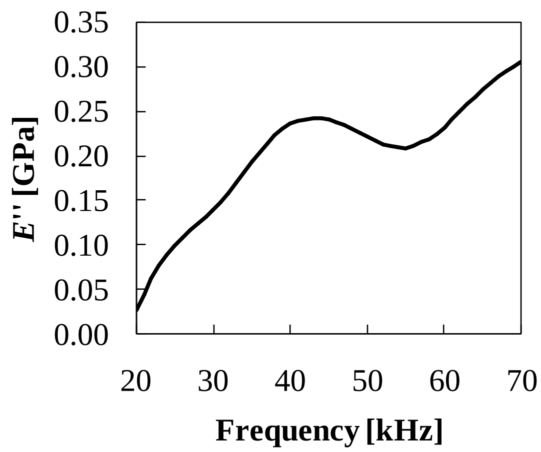
In this study, E^* is estimated from C_g and $\boldsymbol{\alpha}$ by utilizing inverse analysis. We searched suitable E^* to minimize the error e in the following equation.

$$e_{cg,\alpha}(\omega, E^*(\omega)) = (1-w_\alpha)\left|\frac{c_{g,M}(\omega) - c_{g,C}(\omega, E^*(\omega))}{c_{g,M}(\omega)}\right| + w_\alpha\left|\frac{\alpha_M(\omega) - \alpha_C(\omega, E^*(\omega))}{\alpha_M(\omega)}\right| \tag{6.1}$$

Here, $C_{g,M}$ and $\boldsymbol{\alpha_M}$ are measured group velocity and attenuation ratio. For the $C_{g,M}$, the average value of C_g in Fig. 5 is used for the calculation. $C_{g,C}$ and $\boldsymbol{\alpha_C}$ are calculated group velocity and attenuation from E^*. w_α is weight factor for group velocity and attenuation factor. In this study, we set w_α as 0.5.

Estimated complex modulus of elasticity is shown in Fig. 7. Fig. (a) shows real part of the modulus and (b) for imaginary part. Figure 8 is calculated group velocity $C_{g,C}$ and attenuation ratio $\boldsymbol{\alpha_C}$ by using estimated E^*. Measured values of $C_{g,M}$ and $\boldsymbol{\alpha_M}$ are also plotted for the comparison. The fine agreement of calculated and measured data in Fig. 8 shows that inverse analysis performed correctly.

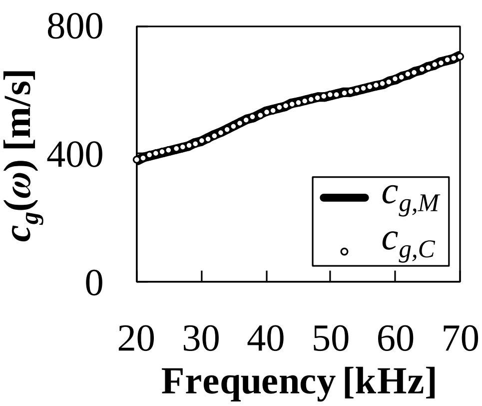


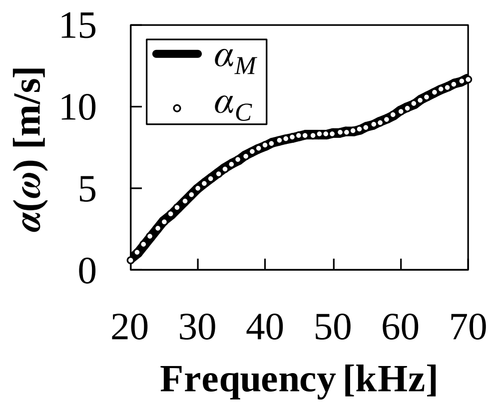


(a) Real part

(b) Imaginary part

Fig. 7. Estimated values of complex modulus $E^*(\omega)$.





(a) Group velocities $c_g(\omega)$

(b) Attenuation ratios $\alpha(\omega)$

Fig. 8. Measured and calculated values of group velocities $c_g(\omega)$ and attenuation ratios $\alpha(\omega)$.

6. Conclusions

In this study, accurate measurement method for both the group velocity and the attenuation ratio for Lamb waves are proposed. For the group velocity measurement, source location algorithm used in acoustic emission monitoring fields is utilized to determine accurate propagation length of Lamb waves. On the other hand, differences in sensitivity of AE sensors are taken into account for the attenuation ratio measurement. Finally, viscoelastic properties of 0.4 mm-thickness PMMA plate are estimated from measured group velocities and attenuation ratios by inverse analysis.

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